An Analysis of the Content and Difficulty of the CSAP 10th-Grade Mathematics Test

A report to the
Denver Area School Superintendents' Council (DASSC)

Lorrie A. Shepard
Dominic D. Peressini
University of Colorado at Boulder

with
Jeffrey A. Frykholm
David R. Grant
Damian Betebenner
University of Colorado at Boulder

William Briggs
William Juraschek
University of Colorado at Denver

Lew Romagnano
James Loats
Metropolitan State College of Denver

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http://education.colorado.edu/EPIC/coloradostudiesandreports.htm

An Analysis of the Content and Difficulty of the CSAP 10th-Grade Mathematics Test

Executive Summary

The present study was undertaken at the request of the Denver Area School Superintendents' Council (DASSC) to provide a more complete understanding of the content and difficulty of the CSAP 10^{th}-grade mathematics test.

The study has three components:

- A content analysis comparing the grade-level content of CSAP to that of other well-known national and international mathematics tests.
- A comparison of CSAP proficiency levels to percentile ranks on nationally-normed tests.
- A comparison of CSAP results to high school course-taking patterns.

Key findings:

➢ The 10^{th}-grade Mathematics CSAP is a difficult test.

- The 10^{th}-grade test is well aligned with the Colorado Model Content Standards.
- 31% of the items in the test require knowledge of mathematics not taught traditionally until after 10^{th}-grade geometry.
- Based on course-content level, the CSAP was found to be substantially more difficult than both the SAT and the 12^{th}-grade TIMSS test, and more difficult than the ACT.

➢ CSAP has very high cut-points for determining partially proficient, proficient, and advanced performance.

- The cutoff for advanced performance is at the 99^{th} percentile of national norms on ACT's PLAN test.
- The cutoff for proficient is at or above the 90^{th} percentile of national norms on PLAN.
- The cutoff for partially proficient is at the 58^{th} percentile of national norms on PLAN.
- Many of the 10^{th}-grade students identified as unsatisfactory on CSAP are above average on the nationally-normed PLAN.

➢ More coursework in mathematics by 10^{th} grade is associated with better performance on CSAP.

Recommendations:

1. Press releases and media reports should acknowledge how high the bar was set when interpreting test results.
2. When rescaling the math tests across grade levels, CDE should use external validity evidence to evaluate whether the proficiency cut-scores at grade 10 were set at appropriate levels.
3. To improve mathematics achievement, districts should examine their curricula in light of the content standards rather than the test and should not hastily abandon reform-based programs.

Policy makers and parents should not jump to conclusions about why Colorado students performed "poorly" on CSAP. It is important to recognize what students can do, as well as what they cannot do, to know where improvement is needed. "Percent Proficient" rates on CSAP do not tell the whole story.

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Background

Senate Bill 186, passed in spring 2000, greatly expanded the Colorado State Assessment Program (CSAP) to include testing of all students in grades 3 through 10 in reading and writing and from grades 5 through 10 in mathematics. The 10^{th}-grade CSAP in mathematics was administered for the first time in spring 2001 and yielded surprising results. Despite the above average performance of Colorado 4^{th} and 8^{th} graders on the National Assessment of Educational Progress, only 14 percent of Colorado 10^{th} graders scored proficient or advanced on CSAP. The Colorado Department of Education press release (July 25, 2001) noted that "results for the Colorado math assessments follow a pattern similar to that seen in many other states as they implement standards based education and the corresponding assessments" (p. 6).

Interpretations of the 10^{th}-grade math results by the media were less sanguine. News headlines tended to focus on the dismal showing in 10^{th} grade math, rather than on other test score gains emphasized by Commissioner Maloney. (Gains were made on seven of eight tests with prior data).

> "Students can't do the math"
> Gazette, Colorado Springs
> "Math portion trips most 10^{th}-graders"
> The Denver Post, July 26, 2001
> "86% miss 10^{th}-grade math mark
> But new CSAP highs reach in 4^{th}-grade reading, writing"
> The Denver Post, July 26, 2001
> "Math test boggles minds"
> Rocky Mountain News, July 26, 2001

A number of mathematics educators were not surprised by the results, given that CSAP requires students to "explain their solution and either present some graphical data or a table or a drawing as part of the explanation," (Flexer quoted in Whaley and Bingham, July 26, 2001). Moreover, the test is 30% algebra, 25% data analysis and probability, and 25% geometry and measurement. Many students are not required to have taken these subjects by 10^{th} grade, if ever, or may be only part way through a relevant course when the test is given in March. Nonetheless, some editorials questioned the validity of the 10^{th}-grade test especially given other evidence that many more than 14% of Colorado's high school students appear to perform well in mathematics.

In the months following the test-score release, school districts have worked to address curricular changes that may be needed to ensure student mastery of challenging mathematical content. The response to low scores in some communities, however, has been to move away from reform-based curricula and to reinstate more traditional mathematics courses. Given that reform-based curricula have more of the content required on CSAP than traditional courses do, such a response is self-defeating. It also indicates a lack of understanding of the Colorado Model Content Standards, CSAP test content, and CSAP performance standards.

What should policy makers and parents conclude from the 14% “pass rate” on the 10^{th}-grade math test? What changes in curriculum or testing should be considered to respond to the test results?

Overview of study purpose and design

The present study was undertaken at the request of the Denver Area School Superintendents’ Council (DASSC) to address the above questions and to provide a more complete understanding of the content and difficulty of the CSAP 10^{th}-grade mathematics test. Once CSAP results are better understood in light of both the challenging content and stringent proficiency standard of the 10^{th}- grade test, then more informed decisions can be made about how to make improvements in curriculum and instruction.

This study has three components:

- A content analysis comparing the grade-level content of CSAP to that of other well-known national and international mathematics tests.
- A comparison of CSAP proficiency levels to percentile ranks on nationally-normed tests.
- A comparison of CSAP results to high school course-taking patterns.

Part 1. Comparison of CSAP Content and Grade Level with SAT, ACT, and TIMSS

CSAP content

The Assessment Frameworks used to develop CSAP tests in each subject area and at each grade level are based on the Colorado Model Content Standards. The six Colorado content standards for mathematics are shown in Figure 1. These broadly-stated standards are intended to apply across grade levels and reflect curricular goals for the use of numbers and number relationships, algebraic methods, data analysis and probability, geometry, measurement, and linking concepts and procedures. The complete Tenth Grade Mathematics Assessment Framework is provided as Appendix A; it shows in detail the kinds of things 10^{th}-grade students should be able to do under each of the standards.

For our analysis of CSAP content, we used the six content standards from the Assessment Framework. (We also had available the complete classification of CSAP items by content subcategories provided by CDE.) In addition, to help lay audiences evaluate whether test content is very challenging or not at all challenging, we developed a continuum representing traditional mathematics course content and expectations. The course sequence continuum ranged from middle school mathematics to Calculus. Thus, each item on the test could be classified on two dimensions. This classification allowed us to describe the mathematical sophistication of each item as well as the content standard assessed.

Figure 1

1. **Students develop number sense* and use numbers and number relationships in problem-solving situations* and communicate the reasoning used in solving these problems.**

2. **Students use algebraic methods* to explore, model*, and describe patterns* and functions* involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.**

3. **Students use data collection and analysis, statistics*, and probability* in problem-solving situations and communicate the reasoning used in solving these problems.**

4. **Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems.**

5. **Students use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems.**

6. **Students link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic*, paper-and-pencil, calculators, and computers, in problem-solving situations and communicate the reasoning used in solving these problems.**

Before proceeding with our analyses, a word of caution is in order. Our use of a traditional course sequence to classify items is not meant as an endorsement of such sequences or as a recommendation that content should be ordered in this way. To the contrary, as we discuss in the course-taking section of the report, students are more likely to gain experience with the communication and problem-solving demands of CSAP, as well as with newer content demands in data analysis and probability, in reform-based curricula than in traditional courses. We chose a traditional sequence for purposes of analysis simply because it is more familiar to policy makers and parents, and because it characterizes the way that mathematics instruction is offered in the majority of high schools in Colorado.

Glenn Bruckhart of the Colorado Department of Education provided an opportunity for the research team to review, under secure conditions, the complete 10th-grade Mathematics CSAP administered in spring 2001. Two professors of mathematics education and four professors of mathematics from the University of Colorado at Boulder, the University of Colorado at Denver, and Metropolitan State College of Denver used a consensus process to classify all of the CSAP items from the actual 2001 assessment instrument by content standard and course level. Table 1 shows the percentage of items classified in each cell of the content matrix. Items worth two, three, or four points in scoring were weighted by the number of points. Thus, the 60-item test actually produced a total of 87 possible points. Where appropriate, different parts of multi-part items were assigned to different categories.

The column totals in Table 1 reflect the relative emphasis given to each of the content standards within the 10th-grade CSAP. These totals correspond closely to the classifications made by test developers. The standards classifications made here differ from those reported by CDE on only six items. University professors counted fewer items in the Computation and Concepts category. Nearly all CSAP items require computation; therefore, it was a matter of degree whether the focus of an item was primarily computational.

For purposes of this study, the most important findings are the summary totals at the far right of the table. We have combined the percent of items from elementary and middle school mathematics with those from Algebra I, because these courses would have been taken by the majority of students prior to 10th grade. Altogether 46% of the items on CSAP reflect content covered in typical middle school and Algebra I classes. Geometry is kept in a category by itself because this is the class that many students are taking during 10th grade, when CSAP is administered. Twenty-two percent of CSAP items require reasoning with and using geometry knowledge and procedures. The remaining 31% of CSAP items require knowledge of content that goes beyond what would have typically been taught in college-preparatory mathematics courses before or during 10th grade. Specifically, 12% of the test items are classified as Algebra II content. The largest group of items not covered in traditional courses, representing 17% of the test content, is the data analysis, statistics, and probability items assigned to the "other" category. Traditionally, students would have been exposed to this content only if they took an elective course in statistics. Thus, many students taking 10th grade CSAP would not have been exposed to 31% of the content knowledge required. For students taking Geometry in 10th grade, an even greater percentage of the content would be unfamiliar by the time of the March test. Students taking more integrated, reform-based mathematics courses would have had more of an opportunity to learn CSAP content by 10th grade, especially the 17% of score points in data analysis and probability (roughly half of the 31% listed as beyond geometry in traditional course sequences).

Table 1
Percentage of Items* on CSAP 10th-Grade Mathematics by Content Standard and Traditional Course Category

<table>
<tr><th colspan="2"></th><th colspan="7">Content Standard</th></tr>
<tr><th colspan="2"></th><th>Number Sense</th><th>Algebra</th><th>Data Analysis and Probability</th><th>Geometry</th><th>Measurement</th><th>Computation and Concepts</th><th>Total</th></tr>
<tr><th rowspan="8">Traditional Course Sequence</th><th>Middle School Mathematics</th><td>8%</td><td>6%</td><td>5%</td><td></td><td>5%</td><td>1%</td><td rowspan="2">46%</td></tr>
<tr><th>Algebra I</th><td>3%</td><td>16%</td><td></td><td></td><td></td><td>2%</td></tr>
<tr><th>Geometry</th><td></td><td></td><td></td><td>15%</td><td>6%</td><td>1%</td><td>22%</td></tr>
<tr><th>Algebra II</th><td></td><td>9%</td><td>3%</td><td></td><td></td><td></td><td rowspan="4">31%</td></tr>
<tr><th>Trigonometry and Pre-Calculus</th><td></td><td>1%</td><td></td><td>1%</td><td></td><td></td></tr>
<tr><th>Calculus</th><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><th>Other</th><td></td><td></td><td>17%</td><td></td><td></td><td></td></tr>
<tr><th>Total</th><td>11%</td><td>32%</td><td>25%</td><td>16%</td><td>11%</td><td>4%</td><td>100%</td></tr>
</table>

*Items worth more than one point are weighted by the number of points.

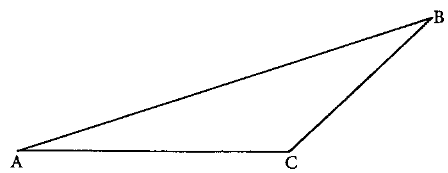
Figure 2A

CSAP Grade 10 Mathematics

Parts A-B Standard: Measurement, Course: Middle School Mathematics
Parts C-D Standard: Geometry, Course: Geometry

2

Use your punch-out ruler and protractor to help you solve this problem. Study the triangle below.



Part A Measure the length of each side of triangle ABC to the nearest tenth of a centimeter. On the lines below, record the length of each side.

AB= _________ centimeters

BC= _________ centimeters

AC = _________ centimeters

Part B Measure angle B and angle C to the nearest degree. On the lines below, record the measurement of each angle.

m∠B = ______ degrees

m∠C = ______ degrees

Part C In the space below, draw a triangle that is larger than, but similar to, Triangle ABC. The scale factor between the sides of Triangle ABC and the new triangle should be 1.5.

Part D On the lines below, explain how you know the new triangle is similar to triangle ABC.

____

____

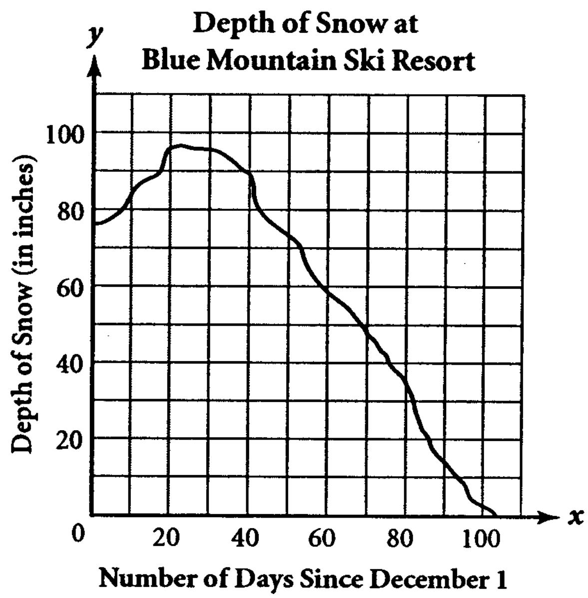
Figure 2B

CSAP Grade 10 Mathematics

Standard: Algebra, Course: Algebra I

4

The graph below shows the depth of snow at Blue Mountain Ski Resort last winter.



Part A On the lines below, explain what the *y*-intercept of the graph represents.

____

____

____

Part B On the lines below, explain what the *x*-intercept of the graph represents.

____

____

____

Figure 2C

CSAP Grade 10 Mathematics

Standard: Data Analysis, Course: Algebra II

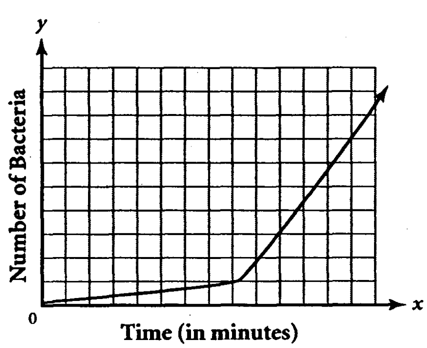
6

The table shows the number of bacteria present at 30 minute intervals during a science experiment.

Number of Bacteria Over Time

Time (in	Number of
0	3
30	6
60	12
90	24
120	48
150	96
180	192
210	384

Which of these graphs best shows the relationship between time and the number of bacteria present?



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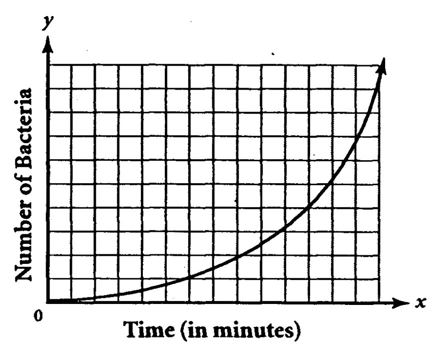
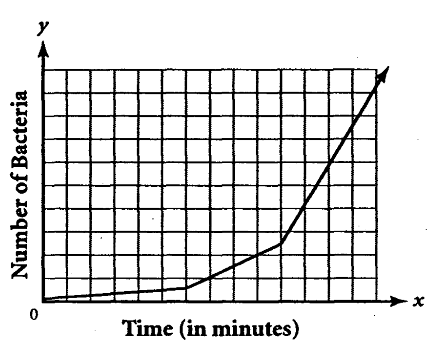


●



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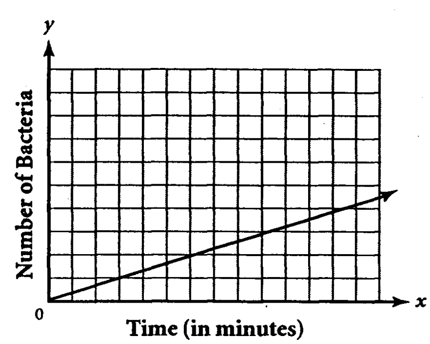


○

Figure 2d

CSAP Grade 10 Mathematics

Standard: Data Analysis, Course: Middle School Mathematics

10

Edgar earned the following scores on his first 10 science tests.

73, 86, 91, 87, 88, 79, 82, 93, 90, 86

Which of these will be affected if Edgar earns a score of 50 on his next test?

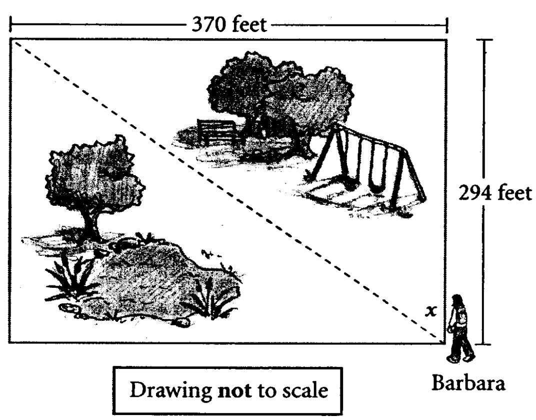
- ❑ mean, median, and mode
- ❑ mean and median
- ❑ mean only
- ❑ median only

Figure 2e

Standard: Geometry, Course: Trigonometry

11

Barbara went for a walk in the city park. To cut across the rectangular park, she chose the path shown by the dotted line in the drawing below.



. At what angle, *x*, did Barbara cut across the park? Round the answer to the nearest tenth of a degree.

- ❑ 37.4
- ❑ 38.5
- ❑ 51.5
- ❑ 52.6

During the item classification process, mathematics experts also noted that a substantial number of CSAP items required reasoning and problem solving abilities that went beyond what would have been expected in traditional high school courses. Items of this type accounted for about one-quarter of all the test score points. Unfortunately, most of these items remain secure and cannot be presented for illustrative purposes. We are able, however, to use released items in Figures 2a and 2b as examples of the kinds of items where students were frequently asked to explain their problem solutions.

The Colorado Department of Education provides detailed information on the content assessed by each test item and the relative difficulty of each item for Colorado 10th graders. To illustrate the range of difficulty of CSAP items we have selected a few of the easiest as well as the most difficult released items from the test. Item 2 (Figure 2a) has four parts and assesses middle school measurement skills as well as knowledge taught in a traditional geometry course. Parts A and B represent some of the easiest content on CSAP and are located in the "Unsatisfactory" range of the score scale, whereas successfully completing parts C and D was much more difficult and are located in the "Proficient" range of the score scale. Another example of a relatively easy item on CSAP is item 10 (Figure 2d), a typical middle school data analysis question.

Released Item 11 (Figure 2d) requires knowledge of trigonometry and is the only released item located in the "Advanced" range. Only seven item points on the test scaled higher than this item and all but one of these required students to explain their answers as well as solve relatively difficult problems. Released items 4 (Figure 2b) and 6 (Figure 2c) are also difficult and were located in the "Proficient" range. Item 4 requires only basic algebra knowledge but illustrates the kind of application and communication skills that make CSAP items more demanding than typical multiple-choice test questions. Item 6 was classified as an Algebra II problem because students in Algebra I typically would not have much experience connecting a non-linear table of values to a graph, although students could use basic graphing skills to solve the problem.

SAT content

Released forms of the SAT are available in 10 Real SATs published by the College Board. To enable comparison of the SAT with the 10th-grade CSAP, items on one complete SAT mathematics test were classified using the same Colorado Model Content Standards and course level matrix as above. Results of this analysis are shown in Table 2. Consistent with the design of the SAT, none of the problems on the math sections require knowledge beyond basic algebra and geometry. The SAT is intended to assess how well students can reason with and use basic mathematics skills, it is not a measure of achievement in advanced mathematics.

Illustrative SAT mathematics items are reprinted in Figure 3 with permission of the College Entrance Examination Board. Thirty-two percent of the items call for basic arithmetic or middle school mathematics skills such as ratio and proportion; percents; averages; factors and multiples; and rate, time, and distance problems. Items 1, 7, and 8, in Figure 3 are examples. An additional 41% of the items require skills typically covered in Algebra I as illustrated by items 4, 6, and 9. Most of these items require sufficient reasoning to be consistent with the 10th-grade standards described in the Assessment Framework (Appendix A). However, 16% of the items were too elementary to be classified in one of the standards categories and are therefore represented in a separate column.

Altogether 73% of SAT content requires skills that college-preparatory students would have covered before 10th grade. The remaining 27% of items require knowledge of geometry covered in 9th or 10th-grade geometry courses, as shown in problems 2, 3, 5, 10, and 11. By the time, students take the SAT in their junior year of high school, they are expected to have had preparation for all of the test content.

Table 2
Percentage of Items on the SAT I Mathematics Test by Content Standard and Traditional Course Category

<table>
<tr><th></th><th></th><th colspan="8">Content Standard</th></tr>
<tr><th></th><th></th><th>Number Sense</th><th>Algebra</th><th>Data Analysis and Probability</th><th>Geometry</th><th>Measurement</th><th>Computation and Concepts</th><th>Too elementary to be classified</th><th>Total</th></tr>
<tr><td rowspan="8">Traditional Course Sequence</td><td>Middle School Mathematics</td><td>13%</td><td></td><td>2%</td><td>2%</td><td></td><td>2%</td><td>13%</td><td rowspan="2">73%</td></tr>
<tr><td>Algebra I</td><td></td><td>25%</td><td>8%</td><td></td><td></td><td>5%</td><td>3%</td></tr>
<tr><td>Geometry</td><td></td><td></td><td></td><td>27%</td><td></td><td></td><td></td><td>27%</td></tr>
<tr><td>Algebra II</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td rowspan="4">0%</td></tr>
<tr><td>Trigonometry and Pre-Calculus</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Calculus</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Other</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Total</td><td>13%</td><td>25%</td><td>10%</td><td>29%</td><td>0%</td><td>7%</td><td>16%</td><td>100%</td></tr>
</table>

Figure 3

SAT sample items

SUMMARY DIRECTIONS FOR COMPARISON QUESTIONS

Answer: A if the quantity in Column A is greater;
B if the quantity in Column B is greater;
C if the two quantities are equal;
D if the relationship cannot be determined from the information given.

1. Middle School Mathematics

Column A	Column B
60% of r	30% of s

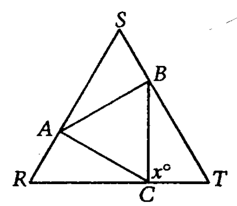
$r = 2s$
$r > 0$

2. Geometry

Column A	Column B
The total surface area of a box with dimensions 3 inches by 3 inches by 6 inches	The sum of the total surface areas of two cubes, each with dimensions 3 inches by 3 inches by 3 inches

3. Geometry

Column A	Column B
x	90



Note: Figure not drawn to scale.

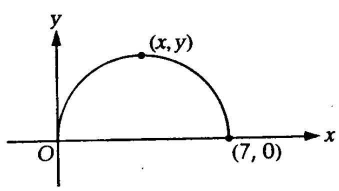
Triangles ABC and RST are each equilateral.

4. Algebra

$n + p + v = 50$
$n + p - v = 20$

Column A	Column B
v	15

5. Geometry



In the figure above, what is the y-coordinate of the point on the semicircle that is the farthest from the x-axis?

6. Algebra

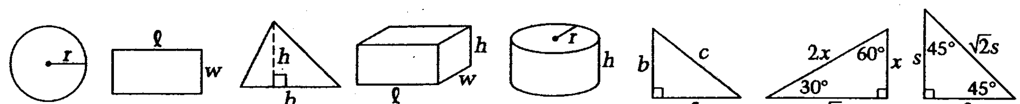
$(T \times 3^3) + (U \times 3^2) + (V \times 3) + W = 50$

Each letter in the equation above represents a digit that is less than or equal to 2. What four-digit number does $TUVW$ represent?

Notes:

1. The use of a calculator is permitted. All numbers used are real numbers.
2. Figures that accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that the figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

Reference Information



$A = \pi r^2$, $C = 2\pi r$ $\quad A = \ell w$ $\quad A = \frac{1}{2}bh$ $\quad V = \ell wh$ $\quad V = \pi r^2 h$ $\quad c^2 = a^2 + b^2$ $\quad$ Special Right Triangles

The number of degrees of arc in a circle is 360.
The measure in degrees of a straight angle is 180.
The sum of the measures in degrees of the angles of a triangle is 180.

7. Middle School Mathematics

If $\frac{x}{9} = \frac{2}{3}$, then $x =$

(A) $\frac{8}{3}$

(B) 3

(C) 6

(D) 7

(E) $\frac{27}{2}$

8. Middle School Mathematics

A total of 60 advertisements were sold for a school yearbook. If 20 percent of the first 20 sold were in color, 40 percent of the next 30 sold were in color, and 80 percent of the last 10 sold were in color, what percent of the 60 advertisements were in color?

(A) 30%

(B) $33\frac{1}{3}$%

(C) 40%

(D) $46\frac{2}{3}$%

(E) 60%

9. Algebra

For which of the following ordered pairs (s, t) is $s + t > 2$ and $s - t < -3$?

(A) (3, 2)

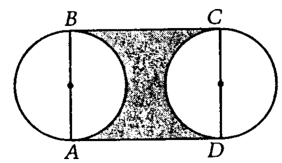
(B) (2, 3)

(C) (1, 8)

(D) $\left(\frac{1}{2}, \frac{3}{2}\right)$

(E) (0, 3)

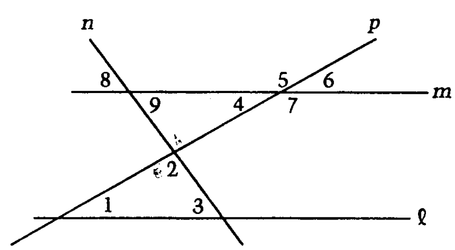
10. Geometry



In rectangle $ABCD$ shown above, sides AB and CD pass through the centers of the two circles. If $AB = 12$ and $AD = 16$, what is the area of the shaded region?

(A) 120
(B) 156
(C) 192
(D) $192 - 36\pi$
(E) $192 - 72\pi$

11. Geometry



In the figure above, line ℓ is parallel to line m. Which of the following pairs of angles have equal measures?

I. 1 and 4
II. 3 and 8
III. 5 and 7

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

ACT content

Content analyses were also completed for the ACT mathematics test using 120 items from two retired forms of the test. Percentages of ACT items falling in each of the standard and course level categories are shown in Table 3 with permission of ACT, Inc.. Unlike the SAT, the ACT requires skills at the Algebra II and Trigonometry level. In fact, the percentages of items in the far right-hand column -- reflecting mathematics skills below geometry, in geometry, and above geometry -- are nearly identical to the corresponding percentages reported for CSAP.

In other respects, however, the ACT mathematics test was judged to be less demanding than the 10^{th}-grade CSAP. Thirteen percent of the items could not be classified according to the content standards because they were too elementary to fit the expectations of the 10^{th}- grade standards described in the Assessment Framework (Appendix A). Sample items in Figure 4 represent the range of item content difficulty found on the ACT, from middle school mathematics to Algebra II and Trigonometry. Sample item 8 is one of the most difficult on the test, and is similar to item 11 on CSAP.

Although it is not possible to accurately evaluate the relative difficulty of CSAP versus ACT without an empirical study that jointly scales the items, it is possible to draw the following rough conclusions based on the judgment of experts. The course-level content of ACT and the 10^{th}-grade CSAP overlap substantially. However, thirteen percent of ACT items are easier than any of the items on CSAP; and between ten and twenty percent of CSAP is more difficult than the ACT because of the requirements that students explain their problem solutions.

Table 3
Percentage of Items on the ACT Mathematics Test by Content Standard and Traditional Course Category

		Content Standard							
		Number Sense	Algebra	Data Analysis and Probability	Geometry	Measurement	Computation and Concepts	Too elementary to be classified	Total
Traditional Course Sequence	Elementary Mathematics	1.7%						0.8%	
	Middle School Mathematics	2.5%	2.5%	2.5%			2.5%	5.0%	
	Algebra I	1.7%	8.3%	0.8%	2.5%		8.3%	5.8%	45.0%
	Geometry				22.5%			0.8%	23.3%
	Algebra II	3.3%	10.0%		2.5%		0.8%		
	Trigonometry and Pre-Calculus	1.7%	5.8%		5.8%			0.8%	
	Calculus								
	Other			0.8%					31.7%
	Total	10.8%	26.7%	4.2%	33.3%		11.7%	13.3%	100.0%

Figure 4.

ACT Mathematics Sample Items

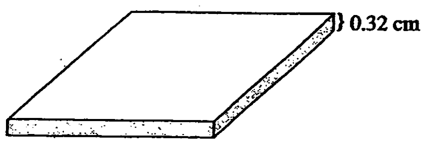
1. Middle School Mathematics

A 20-gram serving of peanut butter contains 9.2 grams of fat. What percent of the weight of this serving is fat?

A. 46%
B. 20%
C. 18.4%
D. 10.8%
E. 9.2%

2. Middle School Mathematics

A computer chip 0.32 cm thick is made up of layers of silicon. If the top and bottom layers are each 0.03 cm thick and the inner layers are each 0.02 cm thick, how many inner layers are there?



F. 13
G. 15
H. 16
J. 52
K. 64

3. Algebra

A bottle of liquid weighs 300 grams when full and 170 grams when half of the liquid is removed. How many grams does the bottle weigh when empty?

F. 10
G. 20
H. 40
J. 85
K. 130

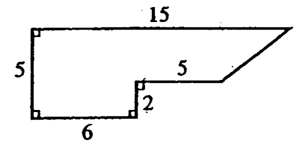
4. Algebra

Which of the following is a factored form of $xy + yx + x$?

F. $x(2y + 1)$
G. $x(y + 1) + y$
H. $3x + 2y$
J. $x^3 + y^2$
K. $y(2x + 1)$

5. Geometry

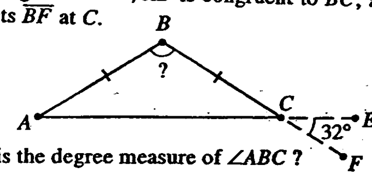
Mr. Gardner wants to put a border around the edges of the flower bed shown in the figure below. Dimensions given on the figure are in feet. What is the distance, in feet, along the edges of the flower bed?



F. 33
G. 38
H. 40
J. 41
K. 58

6. Geometry

In the diagram below, $\overline{AB}$ is congruent to $\overline{BC}$, and $\overline{AE}$ intersects $\overline{BF}$ at C.



What is the degree measure of $\angle ABC$?

A. 26°
B. 32°
C. 64°
D. 116°
E. 148°

7. Algebra II

The floor space in a rectangular room is 14 feet by 17 feet. A circular rug with radius 5 feet lies centered on the floor. If it takes 1 hour to polish the entire floor, which fraction of an hour best estimates the time required to polish only the area of the floor that is NOT covered by the rug?

F. $\frac{238 - 25\pi}{238}$

G. $\frac{238 - 10\pi}{238}$

H. $\frac{62 - 10\pi}{62}$

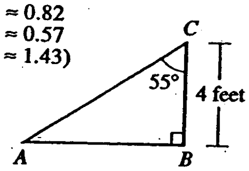
J. $\frac{213}{238}$

K. $\frac{26}{31}$

8. Trigonometry

Approximately how many feet long is $\overline{AB}$ in the right triangle below?

(Note: $\sin 55° \approx 0.82$
$\cos 55° \approx 0.57$
$\tan 55° \approx 1.43$)



A. 2.80
B. 3.28
C. 4.88
D. 5.72
E. 6.97

TIMSS content

The Third International Mathematics and Science Study (TIMSS) is a collaborative research project sponsored by the International Association for the Evaluation of Educational Achievement (IEA). TIMSS reported results for three different populations of students across participating countries: 9-year olds, 13-year olds, and students in their final year of secondary education. (In the U.S., the final-year study sampled high school seniors, while in other countries the final year might include students as old as 19 or 20.)

The TIMSS mathematics literacy test was designed for all final-year students, regardless of their school curriculum. TIMSS also administered an Advanced Mathematics test to students who had completed advanced course work, but it was the mathematics literacy test that was used to rank participating countries and which received so much attention because U.S. students scored below the international average (Mullis, Martin, Beaton, Gonzalez, Kelly, & Smith, 1998).

Released item sets for the final year of secondary school are available at http://timss.bc.edu/TIMSS1/Items.html. Because we did not have complete tests, it was not possible to do an analysis of proportions of items in each content category. However, the set of released items is extensive and also includes empirical information about the relative difficulty of each item for the sample of international students.

Although it may come as a surprise to most readers, the content of the TIMSS math literacy test is remarkably easier than the SAT, ACT, or 10th grade CSAP. *In fact, TIMSS for 12th graders more closely aligns with the content of the 8th-grade CSAP than with the10th-grade CSAP*. To illustrate the basis for this finding, we have paired TIMSS and 8th-grade CSAP items in Appendix B. The first item is an exact match. The ribbon problem appeared on both assessments in identical form except for the change from centimeters to inches. The next pairs show problems requiring similar skills on the two tests: reasoning with multiplication, computing area or volume, estimation, and appropriate interpretation of graphical data. Only the last pair shows items from each of the tests that did not have counterparts on the other.

What this analysis suggests is that U.S. students do not do poorly compared to their international counterparts because they haven't taken enough advanced course work. Rather it suggests that U.S. students do poorly because they lack the conceptual understanding to use and apply mathematics content typically taught before 8th-grade.

This analysis also provides another valuable lesson for interpreting test results. There are two different things that make a test difficult: one is the difficulty of the content, the other is the difficulty of the performance standard defined in terms of the number of items that must be answered correctly. TIMSS has relatively easy test content, but is still difficult for U.S. students in part because they must explain their answers (one-third of TIMSS questions require students to show their work or explain) and because the performance standard (defined in this case as the international norm) is high.

Summary of findings on CSAP content. The content of the CSAP 10th-grade mathematics test is considerably more difficult than the TIMSS mathematics test for 12th graders. In fact, the content of 12th-grade TIMSS maps very closely to the content of the 8th-grade CSAP. CSAP content is also more difficult than the SAT. The content of the SAT is designed not to go beyond geometry, whereas 31% of CSAP requires knowledge typically covered in course work beyond high school geometry. Of the tests examined, the CSAP is more similar to the content demands of the ACT. However, 13% of ACT items were too elementary to be classified according to the 10^{th}-grade standards, and as much as 20% of the CSAP is more challenging than the ACT because students are required to explain their problem solutions.

Part 2. Comparison of CSAP Proficiency Levels to Score Levels on Nationally-Normed Tests

In addition to an analysis of CSAP content, the study also sought to examine the stringency of the performance standards (cut-scores) selected for each of the proficiency levels. In particular, because of its importance in reporting results to the media, we wished to examine the meaning of the passing score for “Proficient” in terms of national norms for high school students.

Four Colorado school districts were able to provide correlated data sets linking students’ results on CSAP with their performance on the PLAN test administered earlier that same year. PLAN is a nationally-normed test developed by the American College Testing Program. Just as the PSAT is the precursor to the SAT, the PLAN closely parallels the content of the ACT and can be used as both a predictor and a practice test for the ACT.

Performance on the two mathematics tests, CSAP and PLAN, is highly correlated. In Boulder Valley, for example, the correlation was .81. This means that the two tests rank order students in roughly the same order. A strong correlation, however, does not necessarily mean that the tests’ interpretive categories align. Data in Tables 4, 5, 6, and 7 show the relationship between CSAP performance categories and PLAN percentiles. For example, in Boulder Valley (Table 4) most students, who were unsatisfactory on CSAP, scored in the middle range on PLAN; 77% of the unsatisfactory students on CSAP scored between the 25^{th} and the 75^{th} percentile nationally. Partially proficient students scored consistently higher. In fact, 72% of partially proficient students scored above the 75^{th} percentile on PLAN. And 72% of proficient students scored above the 95^{th} percentile. All but one advanced student scored at the 99^{th} percentile on PLAN.

Findings from other districts are similar. Colorado students scoring at the partially proficient level on CSAP are not only above average in relation to national norms but for the most part are above the 75^{th} percentile. Specifically, in Cherry Creek 67% of partially proficient students are above the 75^{th} percentile. In Jefferson County, 56% of partially proficient students are above the 75^{th} percentile nationally. And in Douglas County 68% of partially proficient students are above the 75^{th} percentile on the nationally-normed PLAN test.

A technical procedure, called equipercentile equating, was used to obtain a more precise translation from CSAP scale scores to PLAN scores and national percentiles. First, using matched data sets where students had both PLAN and CSAP scores, cumulative

Table 4
Boulder Valley Grade 10 Students' PLAN and CSAP Math Scores

CSAP Math Performance Categories			PLAN Math Percentile Range						
			1-25	**26-50**	**51-75**	**76-85**	**86-95**	**96-99**	**Total**
	US	**Count**	58	123	136	37	5	1	360
		% of US in PLAN range	16.1%	**39.2%**	**37.8%**	10.3%	1.4%	0.3%	100%
	PP	**Count**	4	29	197	302	184	111	827
		% of PP in PLAN range	0.5%	3.5%	23.8%	**36.5%**	22.3%	13.4%	100%
	P	**Count**	1	0	1	28	83	296	409
		% of P in PLAN range	0.2%	0.0%	0.2%	6.9%	20.3%	**72.4%**	100%
	A	**Count**					1	88	89
		% of A in PLAN range					1.1%	**98.9%**	100%
	Total	**Count**	63	152	334	367	273	496	1685
		% of all CSAP in PLAN range	3.7%	9.0%	19.8%	21.8%	16.2%	29.4%	100%

Table 5
Cherry Creek Grade 10 Students' PLAN and CSAP Math Scores

			PLAN Math Percentile Range						
			1-25	**26-50**	**51-75**	**76-85**	**86-95**	**96-99**	**Total**
CSAP Math Performance Categories	**US**	**Count**	156	268	255	42	10	1	732
		% of US in PLAN range	21.3%	**36.6%**	**34.8%**	5.7%	1.4%	.1%	100.0%
	PP	**Count**	5	53	371	467	247	140	1283
		% of PP in Plan Range	.4%	4.1%	28.9%	**36.4%**	19.3%	10.9%	100.0%
	P	**Count**			6	66	157	456	685
		% of P in PLAN range			.9%	9.6%	22.9%	**66.6%**	100.0%
	A	**Count**				1	4	184	189
		% of A in PLAN range				.5%	2.1%	**97.4%**	100.0%
	Total	**Count**	161	321	632	576	418	781	2889
		% of all CSAP in PLAN range	5.6%	11.1%	21.9%	19.9%	14.5%	27.0%	100.0%

Table 6
Jefferson County Grade 10 Students' PLAN and CSAP Math Scores

CSAP Math Performance Categories			PLAN Math Percentile Range						
			1-25	**26-50**	**51-75**	**76-85**	**86-95**	**96-99**	**Total**
	US	**Count**	590	689	565	75	8	3	1930
		% of US in PLAN Range	30.6%	**35.7%**	**29.3%**	3.9%	0.4%	0.1%	100%
	PP	**Count**	32	237	1032	981	462	234	2978
		% of PP in PLAN range	1.1%	8.0%	**34.7%**	**32.9%**	15.5%	7.9%	100%
	P	**Count**		1	12	125	229	518	885
		% of P in PLAN range		0.1%	1.4%	14.1%	25.9%	**58.5%**	100%
	A	**Count**					11	133	144
		% of A in PLAN Range					7.6%	**92.4%**	100%
	Total	**Count**	622	927	1609	1181	710	888	5937
		% of all CSAP in PLAN range	10.5%	15.6%	27.1%	19.9%	12.0%	15.0%	100%

Table 7
Douglas County Grade 10 Students' PLAN and CSAP Math Scores

			PLAN Math Percentile Range				
			1-25	**26-50**	**51-75**	**76-99**	**Total**
CSAP Math Performance Categories	**US**	**% of US in PLAN range**	19%	**32%**	**39%**	10%	100%
	PP	**% of PP in PLAN range**	1%	3%	28%	**68%**	100%
	P	**% of P in PLAN range**		1%	1%	**99%**	100%
	A	**% of A in PLAN range**				**100%**	100%

percentile distributions were calculated for each test. (Note, these were local percentiles based on the particular sample of data and are not the same as national percentiles.) Next, "equal percentiles" from the two tests were paired and plotted in a joint distribution. Equivalent scores can be read from the resulting curve. For example, the CSAP proficient cut score of 551* translates to a PLAN score of 21.4. The final step was to look up the national percentile in the norms conversion table provided by the test publisher. A PLAN score of 21.4 is at the 90th percentile nationally. This analysis was repeated in two other district data sets, where the equipercentile translation for proficient was at the 94th percentile in one case and again at approximately the 90th for the other. An illustration of the results of an equipercentile equating based on one district's data is provided in Appendix C.

Score conversions were also obtained for the partially proficient cut-score and for the advanced cut-score. The CSAP score of 486, which separates unsatisfactory scores from the partially proficient category, is equivalent to a PLAN score of 16, which is at the 58th percentile nationally. Note, then, that this means that Colorado 10th graders are labeled as unsatisfactory unless they score significantly above the national mean. The advanced cut-point of 595 is at the 99th percentile on PLAN.

These very high cut-points, on top of challenging test content, help to explain the "poor performance" of Colorado 10th graders on CSAP. While some part of the shortfall may be real -- in the sense that students have not had experience explaining their answers nor with advanced mathematics topics, it is also the case that the performance standards for proficiency on CSAP were set very high. The math educators who set the proficient cut-point on CSAP apparently wanted students to know a high percentage of the total test content. Such an expectation is consistent with the rhetoric of "world class" standards, i.e., everyone should do as well as world class athletes, but it is not necessarily consistent with how standards are set on all advanced tests. Advanced Placement examinations, for example, have very challenging content but only require that students master about 60% of the content to be given college credit for course mastery. In the case of the Advanced Placement exams, external validity evidence can be gathered to check on the correspondence between the performance of college students and high school students receiving college credit. Without giving up on the high expectations set by the Colorado Model Content Standards, CDE could evaluate the validity of the 10th grade cut-scores by gathering external performance data for a sample of students representing each of the proficiency categories.

Part 3. A comparison of CSAP results to high school course-taking patterns.

To examine the consequences of the finding that the 10th-grade CSAP requires content knowledge beyond what is covered in traditional 10th-grade coursework, districts were also asked to provide data linking student scores to the number and type of mathematics courses taken. These data are summarized in Tables 8, 9, 10, and 11.

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*Note: CSAP scores are reported on a standard score scale, which does not correspond to raw score points.

There is a clear link between the number and level of math courses taken and performance on CSAP. In Boulder Valley (Table 8), for example, the majority of students who were taking no math in 10th grade, who were in a course below the level of Algebra I, or who were in Algebra I scored unsatisfactory on CSAP. The majority of students who were in a 10th-grade Geometry class, 65% in fact, scored in the partially proficient range. Students already in more advanced classes by 10th grade did significantly better. Sixty-one percent of Algebra II students scored as proficient or advanced. Seventy-five percent of the students beyond Algebra II were proficient or advanced. It again speaks to the very high cut-scores on CSAP that the majority of these Algebra II and beyond students were deemed to be proficient rather than advanced. The same pattern is observed in the other districts as well. More coursework in mathematics by 10th grade is associated with better performance on CSAP.

The relationship between CSAP and course taking should be interpreted cautiously because there is, most likely, both a causal and correlational explanation for the link. Given two students with equal prior experience in mathematics, more advanced coursework will very likely improve performance. Most students taking advanced courses, however, also have higher mathematics achievement to start with; therefore, the observed relationship is the result of both course taking and prior achievement. Requiring all students to take geometry, for example, would not automatically improve CSAP scores unless support were also provided to ensure prerequisite knowledge.

Table 8
Boulder Valley Public Schools CSAP Performance Levels for Students With Different Course-Taking Histories

Math Courses Fall Grade 10			CSAP Performance Categories					
			Not Tested	Unsatisfactory	Partially Proficient	Proficient	Advanced	Total
	None	Count	25	43	17	5	2	92
		% within course category	27.2%	**46.7%**	18.5%	5.4%	2.2%	100%
	Below Algebra I	Count	19	86	5	1	0	111
		% within course category	17.1%	**77.5%**	4.5%	0.9%	0.0%	100%
	Algebra I	Count	6	56	25	1	0	88
		% within course category	6.8%	**63.6%**	28.4%	1.1%	0.0%	100%
	Geometry	Count	16	198	576	89	3	882
		% within course category	1.8%	22.5%	**65.3%**	10.1%	0.3%	100%
	Algebra II	Count	7	7	204	275	59	552
		% within course category	1.3%	1.3%	37.0%	**49.8%**	10.7%	100%
	Above Algebra II	Count	5	0	8	43	32	88
		% within course category	5.7%	0.0%	9.1%	**48.9%**	36.4%	100%
		Total Count	78	390	835	414	96	1813

Table 9
Aurora Public Schools CSAP Performance Levels for Students With Different Course-Taking Histories

Number of year-long math courses			CSAP Performance Categories					
			Not Tested	**Unsatisfactory**	**Partially Proficient**	**Proficient**	**Advanced**	**Total**
	0	**Count**	44	88	36	8	2	178
		% within course category	24.7%	**49.4%**	20.2%	4.4%	1.1%	100%
	1	**Count**	29	162	57	3	3	254
		% within course category	11.4%	**63.8%**	22.4%	1.2%	1.2%	100%
	2	**Count**	58	477	458	76	4	1073
		% within course category	5.4%	**44.5%**	**42.7%**	7.1%	0.4%	100%
	3	**Count**	2	2	2			6
		% within course category	33.3%	33.3%	33.3%			100%
		Total Count	133	729	553	87	9	1511

Table 10
Jefferson County School District CSAP Performance Levels for Students With Different Course-Taking Histories

			CSAP Performance Categories				
			Unsatisfactory	Partially Proficient	Proficient	Advanced	Total
Course Selection Path	Below Grade Level	Count	751	223	5	1	980
		% within course category	76.6%	22.8%	0.5%	0.1%	
	On Grade Level	Count	863	2103	306	24	3296
		% within course category	26.2%	63.8%	9.3%	0.7%	
	Above Grade Level	Count	35	499	582	115	1231
		% within course category	2.8%	40.5%	47.3%	9.3%	
		Total Count	1649	2825	893	140	5507

Note: This analysis is comparing three course selection paths to the performance levels of CSAP. It was necessary to group the courses in this manner for the following reasons:

1. Because of the various sequencing of courses in Jefferson County High Schools
2. Because several of our high schools offer Integrated Math or Interactive Math Program (IMP)

Table 11
Douglas County School District CSAP Performance Levels for Students With Different Course-Taking Histories

		CSAP Performance Categories				
		Unsatisfactory	**Partially Proficient**	**Proficient**	**Advanced**	**Total**
No Algebra II or Geometry	**% within course category**	**65%**	31%	1%		100%
Algebra II or Geometry	**% within course category**	22%	**67%**	9%		100%
Algebra II and Geometry	**% within course category**	5%	**44%**	**41%**	9%	100%

Summary and Recommendations

The 10^{th}-grade Mathematics CSAP is a difficult test. The content, designed to align with the Colorado Model Content Standards, is challenging. Thirty-one percent of the test requires knowledge of mathematics not taught traditionally until after 10^{th}-grade geometry. Based on course-content level, the CSAP was found to be substantially more difficult than both the SAT and the TIMSS 12^{th}-grade test. CSAP is more similar to the ACT, in terms of the course-level of the items, but is still more difficult than ACT because students must explain their answers on many CSAP items.

A second source of difficulty on CSAP is the high performance standards that were set for partially proficient, proficient, and advanced. Tests with difficult content -- such as Advanced Placement Exams -- do not always require that students master a high percentage of the content. In the case of CSAP, however, the proficiency levels were set with the expectation that students should master a high proportion of the challenging content. Using matched data sets from three districts it was possible to translate CSAP proficiency cut-scores into equivalent scores on the nationally-normed PLAN test. The cut-off score required to be partially proficient rather than unsatisfactory is at the 58^{th} percentile nationally on the PLAN; to be proficient, one must be at the 90^{th} percentile or above; to be advanced one must be at the 99^{th} percentile.

Recommendations

1. Press releases and media reports should acknowledge how high the bar was set when interpreting test results.

   For example, if a runner were trying to beat the world record and didn't make it, commentators would not conclude that he or she was a poor runner. CDE set the right tone by focusing on progress in implementing reforms, with more work still to be done. Yet, the media has not been discouraged from using "failure" as the way to describe students in the partially proficient category. Better understanding of the proficiency of students in this category, as well as of students in the unsatisfactory range, is needed. The CDE could foster such understanding by providing the media with composite samples of work that characterize the performance of partially proficient and unsatisfactory students.

2. When rescaling the math tests across grade levels, CDE should use external validity evidence to evaluate whether the proficiency cut-scores at grade 10 were set at appropriate levels.

   Measures of advanced achievement (such as AP exams) do not necessarily require that students master a high proportion of a difficult content domain to be deemed proficient. Independent evidence from students who have demonstrated their problem-solving and communication proficiency in mathematics (e.g., using classroom samples of work) could be used to evaluate the validity and reasonableness of the 10th grade cut-scores.

3. To improve mathematics achievement, districts should examine their curricula in light of the content standards rather than the test and should not hastily abandon reform-based programs.

   "Disappointing" results on the 10th-grade CSAP do not imply that districts should abandon reform-based curricula in favor of more traditional course content. Student performance could be improved first and foremost by making sure that students have a good conceptual understanding of the math they are learning, are adept at using it in applied problems, and gain experience in explaining their reasoning when solving a problem. Next most important would be attention to content not taught in traditional mathematics courses, especially concepts in data analysis and data interpretation.

References

Mullis, I. V. S., Martin, M. O., Beaton, A. E., Gonzales, E J., Kelly, D. L., Smith, T. A. (1998). *Mathematics and Science Achievement in the Final Year of Secondary School: IEA's Third International Mathematics and Science Study (TIMSS).* Chestnut Hill, MA: TIMSS International Study Center, Boston College.

Appendix A

Colorado Department of Education

Tenth Grade Mathematics Assessment Framework

http://www.cde.state.co.us/cdeassess/as_g10maframework.htm

INTRODUCTION

With the passage of SB 00-186, Colorado Student Assessment Program has greatly expanded. CSAP will now include assessments in reading and writing in grades 3 through 10, assessments in mathematics in grades 5 through 10, a science assessment at grade 8, and the ACT Assessment at grade 11. Next year we will begin phasing in this new schedule so that in two years all the new assessments will be in place.

	1999 - 2000 GRADES	**2000 - 2001 GRADES**	**2001 -2002 GRADES**
Reading	3, 4, 7	3, 4, 5, 6, 7, 8, 9, 10	Same
Writing	4, 7	4, 7, 10	3, 4, 5, 6, 7, 8, 9, 10
Math	5 (fall), 8	5 (spring), 8, 10	5, 6, 7, 8, 9, 10
Science	8	8	8
ACT		11	11

The process of constructing each of the new CSAP assessments involves teachers and curriculum specialists from around the state. The first step in developing the new assessments in reading, writing, and mathematics was to delineate the knowledge and skills students must possess at each grade level that can be assessed on a state test. To do this, the Colorado Department of Education convened committees comprised of teachers and content area experts at each grade level to develop assessment frameworks, which will guide the development of the new CSAP assessments.

Using the Colorado Model Content Standards and Suggested Grade Level Expectations as their guides, the committees developed draft assessment frameworks. These draft frameworks were then placed on the Colorado Department of Education website to be reviewed by educators throughout the state. Each committee then reconvened to finalize the assessment frameworks by considering all suggestions posed by the external reviewers.

While the assessment frameworks list the knowledge and skills that will be assessed by CSAP assessments at each grade, they are not new standards nor are they curriculum. To understand the role each document plays in Colorado's educational system, it is important to understand the relationship between the Colorado Model Content Standards, the Suggested Grade Level Expectations, and the assessment frameworks.

- The Colorado Model Content Standards represent the fundamental knowledge and skills Coloradoans expect that students should possess at various intervals as they move through their educational careers. These documents were developed with the help of thousands of Coloradoans over the course of two years. The Colorado Model Content Standards form the basis for the standards-based education movement in Colorado.

- The Suggested Grade Level Expectations were derived from the Colorado Model Content Standards. They help define what could be expected of students at each grade level as opposed to grade ranges such as K-4. They were not as extensively reviewed as the Colorado Model Content Standards, and they were not developed to inform test construction.

- The assessment frameworks are derived from the Colorado Model Content Standards as well. They define what can be assessed at each grade level by one particular assessment (CSAP). Because CSAPs are limited to selected-response and constructed-response items, the framework is more limited in scope. The assessment frameworks are not intended to be a new set of standards, but rather, a guide for test construction.

Weighting of Standards by Grade Level for Mathematics CSAP

<table>
<tr><th></th><th>Grade 5</th><th>Grade 6</th><th>Grade 7</th><th>Grade 8</th><th>Grade 9</th><th>Grade 10</th></tr>
<tr><td>Standard</td><td>% Items</td><td>% Items</td><td>% Items</td><td>% Items</td><td>% Items</td><td>% Items</td></tr>
<tr><td>1</td><td>20</td><td>20</td><td rowspan="2">30</td><td rowspan="2">25</td><td rowspan="2">20</td><td rowspan="2">20</td></tr>
<tr><td>6</td><td>20</td><td>15</td></tr>
<tr><td>2</td><td>20</td><td>20</td><td>20</td><td>25</td><td>30</td><td>30</td></tr>
<tr><td>3</td><td>20</td><td>20</td><td>20</td><td>20</td><td>25</td><td>25</td></tr>
<tr><td>4 and 5</td><td>20</td><td>25</td><td>30</td><td>30</td><td>25</td><td>25</td></tr>
</table>

TENTH GRADE ASSESSMENT FRAMEWORK

Standard 1:

Students develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems.

In grade 10, what students know and are able to do includes

1.1 Demonstrating meanings for real numbers*, absolute value*, and scientific notation* using physical materials and technology in problem-solving situations*.

1.1a Compare and order sets of real numbers.

1.1b Recognize and use equivalent representations of real numbers in a variety of forms including scientific notation, radicals, and other irrational numbers* such as π.

1.1c Use very large and very small numbers in real-life situations to solve problems (for example, understanding the size of the national debt).

1.2 Developing, testing, and explaining conjectures* about the properties of number systems and sets of numbers.

1.2a Develop and test conjectures about the properties of the real number system and common subsets of the real number system (for example, counting numbers, integers*, rationals)

1.2b Verify and apply the properties of the operation "to the power of".

1.3 Use number sense to estimate and justify the reasonableness of solutions to problems involving real numbers.

Standard 2:

Students use algebraic methods to explore, model and describe patterns and functions involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.

In grade 10, what students know and are able to do includes

2.1 Modeling real world phenomena (for example, distance-versus-time relationships, compound interest, amortization tables, mortality rates) using functions*, equations, inequalities, and matrices*.

2.1a Model* real world phenomena involving linear, quadratic, and exponential relationships using multiple representations of rules that can take the form of a recursive process, a function, an equation, or an inequality.

2.2 Representing functional relationships using written explanations, tables, equations, and graphs, and describing the connections among these representations.

2.2a Represent functional relationships using written explanations, tables, equations, and graphs and describe the connections among these representations.

2.2b Convert from one representation to another.

2.2c Interpret a graphical representation of a real-world situation.

2.3 Solving problems involving functional relationships using graphing calculators and/or computers as well as appropriate paper-and pencil techniques.

2.3a Solve problems involving functions and relations using calculators, graphs, tables and algebraic methods*.

2.3b Solve simple systems of equations and inequalities using algebraic, graphical, or numeric methods.

2.3c Solve equations with more than one variable* for a given variable (for example, solve for p in $I = prt$ or for r in $c = 2\pi r$)

2.4 Analyzing and explaining the behaviors, transformations*, and general properties of types of equations and functions (for example, linear*, quadratic*, exponential*).

2.4a Identify and interpret x- and y-intercepts in the context of a problem.

2.4b Using a graph, identify the maximum and minimum value within a given domain.

2.4c Demonstrate horizontal and vertical translations on graphs of functions and their meanings in the context of a problem.

2.4d Recognize when a relation is a function.

2.5 Interpreting algebraic equations and inequalities geometrically and describing geometric relationships algebraically.

2.5a Graph solutions to equations and inequalities in one- and two-dimensions.

2.5b Express the perimeter, area and volume* relationships of geometric figures algebraically.

2.5c Describe geometric relationships algebraically.

Standard 3:

Students use data collection and analysis, statistics, and probability in problem-solving situations and communicate the reasoning and processes used in solving these problems.

In grade 10, what students know and are able to do includes

3.1 Designing and conducting a statistical experiment to study a problem, and interpreting and communicating the results using the appropriate technology (for example, graphing calculators, computer software)

3.1a Identify factors which may have affected the outcome of a survey (for example, biased questions or collection methods).

3.1b Draw conclusions about a large population based upon a properly chosen random sample.

3.1c Select and use an appropriate display to represent and describe a set of data (for example, scatter plot*, line graph, histogram).

3.2 Analyzing statistical claims for erroneous conclusions or distortions.

3.2a Check a graph, table, or summary for misleading characteristics.

3.2b Recognize the misuse of statistical data in written arguments.

3.2c Describe how data can be interpreted in more than one way or be used to support more than one position in a debate.

3.2d Describe how the responses to a survey can be affected by the way the questions are phrased and/or by the reader's bias.

3.3 Fitting curves to scatter plots using informal methods or appropriate technology to determine the strength of the relationship between two data sets and to make predictions.

3.3a Graph data sets, create a scatter plot, and identify the control (independent) variable and dependent variable.

3.3b Determine a line of best fit from a scatter plot using visual techniques.

3.3c Predict values using a line of best fit.

3.3d Show how extrapolation may lead to faulty conclusions.

3.3e Recognize which model, linear or nonlinear, fits the data most appropriately.

3.4 Drawing conclusions about distributions of data based on analysis of statistical summaries (for example, the combination of mean and standard deviation, and differences between the mean and median).

3.4a Differentiate between mean, median, and mode and demonstrate the appropriate use of each.

3.4b Recognize and classify various types of distributions (for example, bimodal, skewed, uniform, binomial, normal).

3.4c Use the mean and standard deviation to determine relative positions of data points in a normal distribution of authentic data.

3.4d Demonstrate how outliers might affect various representations of data and measures of central tendency.

3.5 Using experimental and theoretical probability* to represent and solve problems involving uncertainty (for example, the chance of playing professional sports if a student is a successful high school athlete).

3.5a Determine the probability of an identified event using the sample space.

3.5b Distinguish between experimental and theoretical probability and use each appropriately.

3.5c Differentiate between independent and dependent events to calculate the probability in real-world situations.

3.5d Calculate the probability of event A **and** B occurring and the probability of event A **or** B occurring.

3.5e Use area models to determine probability (for example, the probability of hitting the bull's eye region in a target).

3.6 Solving real-world problems* with informal use of combinations* and permutations* (for example, determining the number of possible meals at a restaurant featuring a given number of side dishes).

3.6a Apply organized counting techniques to determine combinations and permutations in problem-solving situations.

Standard 4:

Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems.

In grade 10, what students know and are able to do includes

4.1 Finding and analyzing relationships among geometric figures using transformations (for example, reflections*, translations*, rotations*, dilations*) in coordinate systems*.

4.1a Describe and apply the properties of similar and congruent* figures.

4.1b Solve problems involving symmetry* and transformations.

4.1c Use coordinate geometry* and/or tessellations to solve problems.

4.1d Describe cylinders, cones and spheres that result from the rotation of rectangles, triangles and semicircles about a line.

4.2 Deriving and using methods to measure perimeter, area, and volume of regular and irregular geometric figures.

4.2a Use the Pythagorean Theorem and its converse to solve real-world problems.

4.2b Use properties of polygons to find areas of regular and irregular figures.

4.2c Use properties of geometric solids to find volumes and surface areas of regular and irregular geometric solids.

4.3 Making and testing conjectures about geometric shapes and their properties, incorporating technology where appropriate.

4.3a Make and test conjectures about geometric shapes and their properties to include parallelism and perpendicularity, numerical relationships on a triangle, relationships between triangles, and properties of quadrilaterals and regular polygons.

4.3b Apply geometric relationships such as parallelism and perpendicularity, numerical relationships on a triangle, relationships between triangles, and properties of quadrilaterals and regular polygons to solve problems.

4.4 Using trigonometric ratios* in problem-solving situations (for example, finding the height of a building form a given point, if the distance to the building and the angle of elevation are known).

4.4a Use right triangle trigonometry* to solve real-world problems.

Standard 5:

Student use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning involved in solving these problems.

In grade 10, what students know and are able to do includes

5.1 Measuring quantities indirectly using techniques of algebra*, geometry, or trigonometry.

5.1a Use appropriate measurements to solve problems indirectly (for example, find the height of a flagpole using similar triangles or right triangle trigonometry).

5.1b Use measurement to solve real-world problems involving rate of change (for example, distance traveled using rate and time).

5.1c Given the rate of change, model real-world problems algebraically or graphically.

5.1d Describe how changing the measure of one attribute of a geometric figure affects the other measurements.

5.2 Selecting and using appropriate tools and techniques to measure quantities in order to achieve specified degrees of precision, accuracy, and error (or tolerance) of measurements.

5.2a Select and use appropriate tools and techniques to measure quantities in order to achieve specified degrees of precision, accuracy, and error of measurements.

5.2b Given commonly used multi-dimensional figures, determine what units and measurements need to be taken.

5.3 Determining the degree of accuracy of a measurement (for example, by understanding and using significant digits).

5.3a Determine the number of significant digits when measuring and calculating with those measurements.

Standard 6:

Students link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic, paper-and-pencil, calculators, and computers, in problem-solving situations and communicate the reasoning involved in solving these problems.

In grade 10, what students know and are able to do includes

6.1 Using ratios, proportions, and percents in problem-solving situations.

6.1a Use ratios, proportions, and percent in problem-solving situations that involve rational numbers*.

6.1b Convert from one set of units to another (for example, feet/minute to miles/hour).

6.1c Apply direct variation to problem-solving situations.

6.2 Selecting and using appropriate methods for computing with real numbers in problem-solving situations from among mental arithmetic*, estimation, paper-and-pencil, calculator, and computer methods, and determining whether the results are reasonable.

6.2a Apply appropriate computational methods to solve multi-step problems involving all types of numbers from the real number system.

6.3 Describing the limitations of estimation and assessing the amount of error resulting from estimation within acceptable tolerance limits.

6.3a Determine when estimation is an appropriate method to solve a problem and describe what error might result from estimation.

***Note: The definitions for words designated with an asterisk(*) in this document may be found in the following glossary of the Colorado Model Content Standards for Mathematics.**

GLOSSARY

Absolute value — A number's distance from zero on a number line. The absolute value of -6, shown as |-6|, is 6, and the absolute value of 6, shown as |6|, is 6.

Algebra — The branch of mathematics that is the generalization of the ideas of arithmetic.

Algebraic methods — The use of symbols to represent numbers and signs to represent their relationships.

Combinations — Subsets chosen from a larger set of objects in which the order of the items doesn't matter (for example, the number of different committees of three that can be chosen from a group of twelve members).

Congruent or the concept of **congruence** — Two figures are said to be congruent if they are the same size and shape.

Coordinate geometry — Geometry based on the coordinate system.

Coordinate system (also called rectangular coordinate system) — A method of locating points in the plane or in space by means of numbers. A point in a plane can be located by its distances from both a horizontal and a vertical line called the axes. The horizontal line is called the x-axis. The vertical line is called the y-axis. The pairs of numbers are called ordered pairs. The first number, called the x-coordinate, designates the distance along the horizontal axis. The second number, called the y-coordinate, designates the distance along the vertical axis. The point at which the two axes intersect has the coordinates (0,0) and is called the origin.

Conjecture — A statement that is to be shown true or false. A conjecture is usually developed by examining several specific situations.

Dilation — A transformation that either enlarges or reduces a geometric figure proportionally.

Exponential function — A function that has an equation of the form y= ax. These functions are used to study population growth or decline, radioactive decay, and compound interest.

Function — A relationship between two sets of numbers (or other mathematical objects). Functions can be used to understand how one quantity varies in relation to another, for example, the relationship between the number of cars and the number of tires.

Integers — The set of numbers consisting of the counting numbers (that is, 1, 2, 3, 4, 5, ...), their opposites (that is, negative numbers, -1, -2, -3, ...), and zero.

Irrational numbers — The set of numbers which cannot be represented as fractions. Examples are √2, e, and π.

Linear function — A function that has a constant rate of change.

Matrix (pl. **matrices**) — A rectangular array of numbers (or letters) arranged in rows and columns.

Mental arithmetic — Performing computations in one's head without writing anything down. Mental arithmetic strategies include finding pairs that add up to 10 or 100, doubling, and halving.

Model — To make or construct a physical or mathematical representation.

Permutations — All possible arrangements of a given number of items in which the order of the items makes a difference. For example, the different ways that a set of four books can be placed on a shelf.

Probability — The likeliness or chance of an event occurring.

Problem-solving situations — Contexts in which problems are presented that apply mathematics to practical situations in the real world, or problems that arise from the investigation of mathematical ideas.

Quadratic function — A function that has an equation of the form y = ax2+bx+c, where a≠0. These functions are used to describe the flight of a ball and the stream of water from a fountain.

Rational numbers — A number that can be expressed in the form a/b, where a and b are integers and b≠0, for example, 3/4, 2/1, or 11/3. Every integer is a rational number, since it can be expressed in the form a/b, for example, 5 = 5/1. Rational numbers may be expressed as fractional or decimal numbers, for example, 3/4 or .75. Finite decimals, repeating decimals, and mixed numbers all represent rational numbers.

Real numbers — All rational and irrational numbers.

Real-world problems (also called real-world experiences) — Quantitative problems that arise from a wide variety of human experiences which may take into consideration contributions from various cultures (for example, Mayan or American pioneers), problems from abstract mathematics, and applications to various careers (for example, making change or calculating the sale price of an item).

Reflection (also called a **flip**) — A transformation which produces the mirror image of a geometric figure.

Rotation (also called a **turn**) — A transformation which turns a figure about a point a given number of degrees.

Scatter plot (also called scatter diagram or scattergram) — A graph of the points representing a collection of data.

Scientific notation — A short-hand way of writing very large or very small numbers. A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10, for example, 4.53 X 103 = 4350.

Symmetry — The correspondence in size, form, and arrangement of parts on opposite sides of a plane, line, or point. For example, a figure that has line symmetry has two halves which coincide if folded along its line of symmetry.

Transformation — The process of changing one configuration or expression into another in accordance with a rule. Common geometric transformations include translations, rotations, and reflections.

Translation (also called a **slide**) — A transformation that moves a geometric figure by sliding. Each of the points of the geometric figure moves the same distance in the same direction.

Trigonometric ratios — The ratios of the lengths of pairs of sides in a right triangle. There are three basic trigonometric ratios used in trigonometry: sine (sin), cosine (cos), and tangent (tan).

Trigonometry — A branch of mathematics that combines arithmetic, algebra, and geometry. Trigonometry is used in surveying, navigation, and various sciences such as physics.

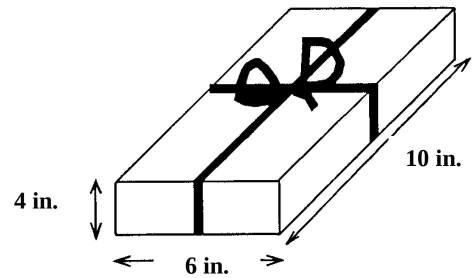
Variable — A quantity that may assume any one of a set of values. In the equation 2x + y = 9, x and y are variables.

Volume — The measure of the interior of a three-dimensional figure. A unit for measuring volume is the cubic unit.

CSAP Grade 8 Mathematics

1

Jeff wants to wrap ribbon around a package, as shown below. He also needs 10 more inches of ribbon to tie a bow.



How much ribbon does he need to wrap the package and to tie the bow?

- ❑ 34 inches
- ❑ 48 inches
- ❑ 50 inches
- ❑ 58 inches

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CSAP Grade 8 Mathematics

2

Alice is looking at a map to see how far it is from her home to the state fair in Pueblo. The scale on the map is 1 inch = 125 miles. The distance between her home and the state fair is 1 ½ inches on the map.

What is the actual distance between Alice's home and the state fair?

- ❑ 83 ½ miles
- ❑ 175 ½ miles
- ❑ 187 ½ miles
- ❑ 250 miles

TIMSS Mathematics Literacy – Final Year of Secondary School

D8.

In a vineyard there are 210 rows of vines. Each row is 192 m long and plants are planted 4 m apart. On average, each plant produces 9 kg of grapes each season.

The total amount of grapes produced by the vineyard each season is closest to

A. 10 000 kg

B. 100 000 kg

C. 400 000 kg

D. 1 600 000 kg

CSAP Grade 8 Mathematics

5

Kim is making pizza for the school carnival. If the radius of the pizza is doubled, how will the area change?

- ❑ The area will remain the same.
- ❑ The area will be two times as large.
- ❑ The area will be three times as large.
- ❑ The area will be four times as large.

TIMSS Mathematics Literacy – Final Year of Secondary School

D12.

Brighto soap powder is packed in cube-shaped cartons. A carton measures 10 cm on each side.

The company decides to increase the length of each edge of the carton by 10 per cent.

How much does the volume increase?

A. 10 cm^3

B. 21 cm^3

C. 100 cm^3

D. 331 cm^3

CSAP Grade 8 Mathematics

8

Elaine is shopping for new office supplies. She has made a list of the items she will purchase.

```
Computer paper $29.25
Hole punch $10.99
Calculator $89.99
Folders $14.49
```

Estimate how much Elaine will spend. In the space below, explain or show your work and write your answer on the line.

Estimate:______________________________

TIMSS Mathematics Literacy – Final Year of Secondary School

D16.

Teresa wants to record 5 songs on tape. The length of time each song plays for is shown in the table.

Song	Length of Time
1	2 minutes 41 seconds
2	3 minutes 10 seconds
3	2 minutes 51 seconds
4	3 minutes
5	3 minutes 32 seconds

Estimate to the nearest minute the total time taken for all five songs to play and explain how this estimate was made.

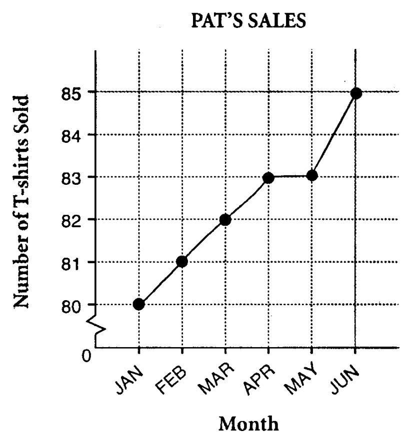
Estimate: ______________

Explain:

CSAP Grade 8 Mathematics

14

Pat was planning to ask her boss for a raise. She made the graph below to show her boss the increase in her T-shirt sales.



Her boss said that the graph was misleading and that Pat's sales did not improve very much. On the lines below, explain how the graph is misleading.

____

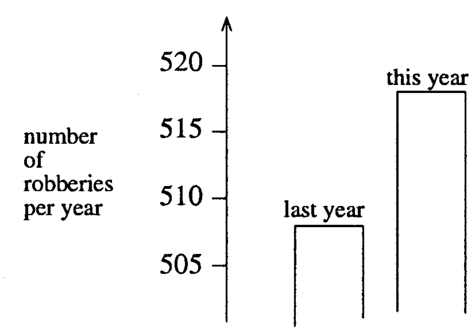
____

TIMSS Mathematics Literacy – Final Year of Secondary School

D17.

A TV reporter showed this graph and said:

"There's been a huge increase in the number of robberies this year."



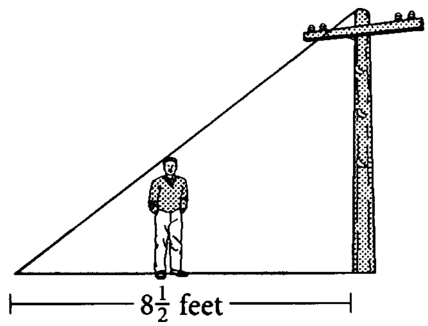
Do you consider the reporter's statement to be a reasonable interpretation of the graph? Briefly explain.

CSAP Grade 8 Mathematics

(problem with no counterpart on TIMSS)

7

Lloyd is standing near a telephone pole as shown in the figure below. When his head touches the support wire, he is 2 ½ feet from where the wire meets the ground. If Lloyd is 5 feet tall, how tall is the telephone pole?



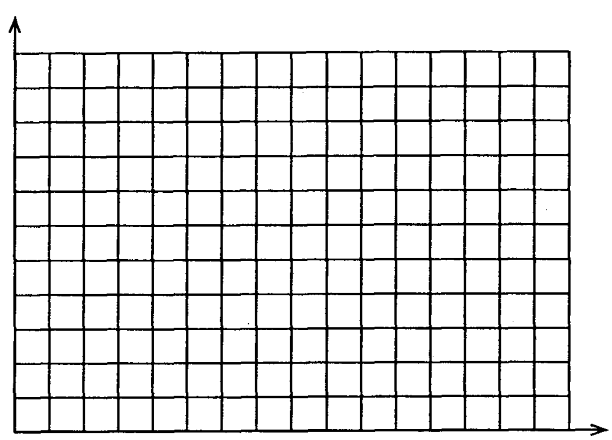
- ❑ 15 feet
- ❑ 17 feet
- ❑ 20 feet
- ❑ 80 feet

TIMSS Mathematics Literacy – Final Year of Secondary School

(problem with no counterpart on CSAP)

A10.

Using the set of axes below, sketch a graph which shows the relationship between the height of a person and his/her age from birth to 30 years. Be sure to label your graph, and include a realistic scale on each axis.



Appendix C

Equipercentile Equating of CSAP and PLAN Scores Using Cherry Creek Data